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Title:	Quantifying the Collective Gain in Monitoring Capability Achieved through DNE23: A High Level, Agent Based Model.
Author(s):	Carmichael, Joshua Daniel
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Quantifying the Collective Gain in Monitoring Capability Achieved through DNE23

A High Level, Agent-Based Model

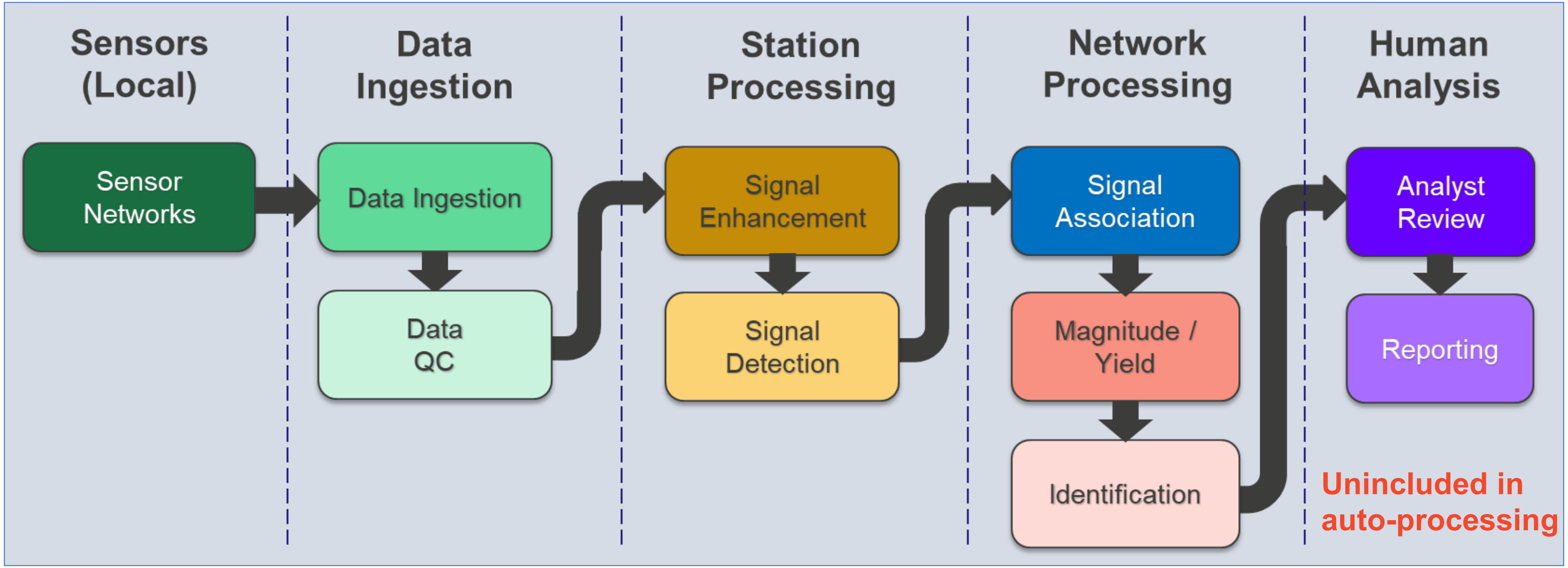
Joshua D Carmichael

04-March-2021

The Event Processing Pipeline (EPP) Model Discussion

- **Chapeau Question 10:** "How has low yield monitoring capability been improved through results of this [the LYNM-DN] program?" demonstrates that we need to quantify our overall gain in performance.
- **Measure changes in performance:** Offer a general, signature-agnostic model that measures relative gains sourced by each FA to gains in the event processing pipeline, over a baseline.
- **Three typologies of change**: agent, workflow topology, and agent burdens

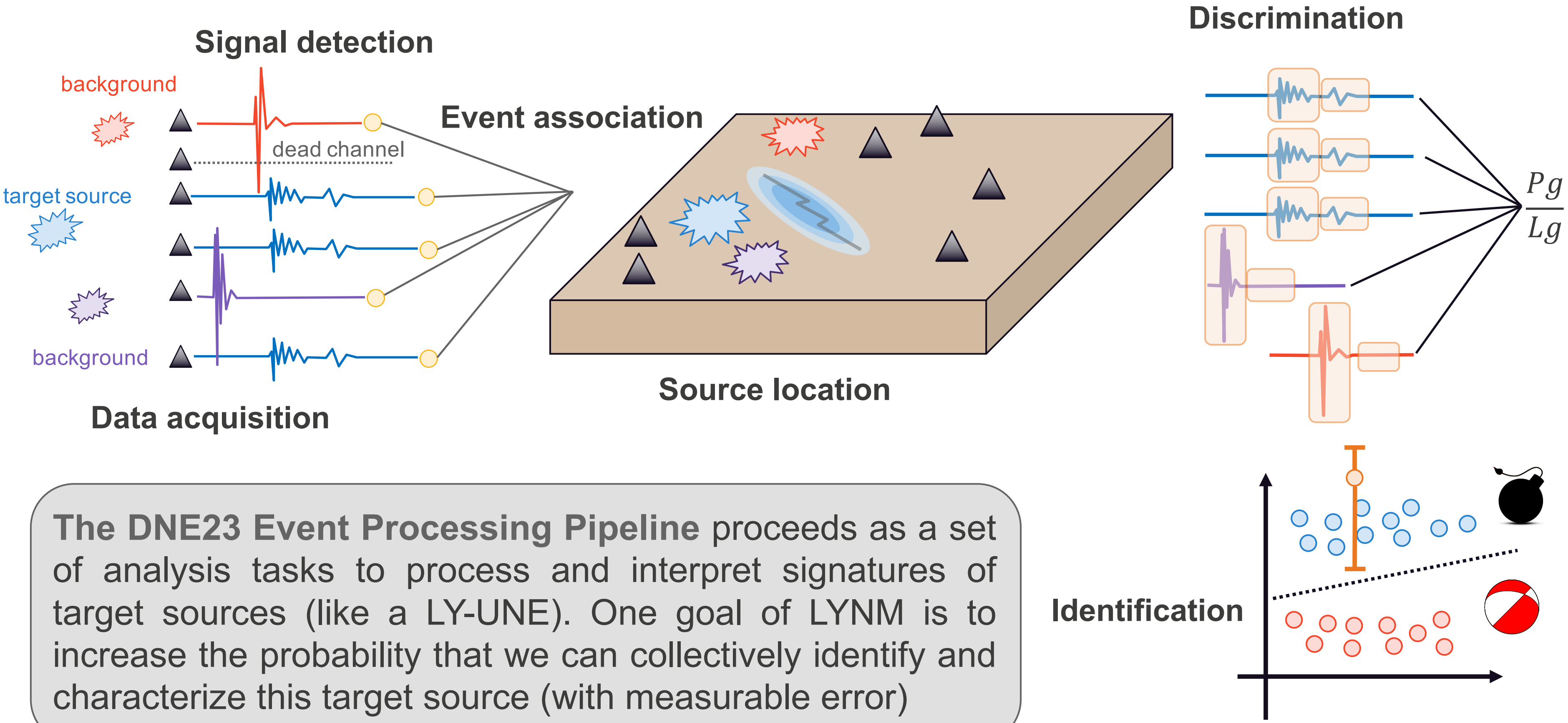
The problem



Conceptual Model for DNE23 / Event Processing Pipeline Workflow

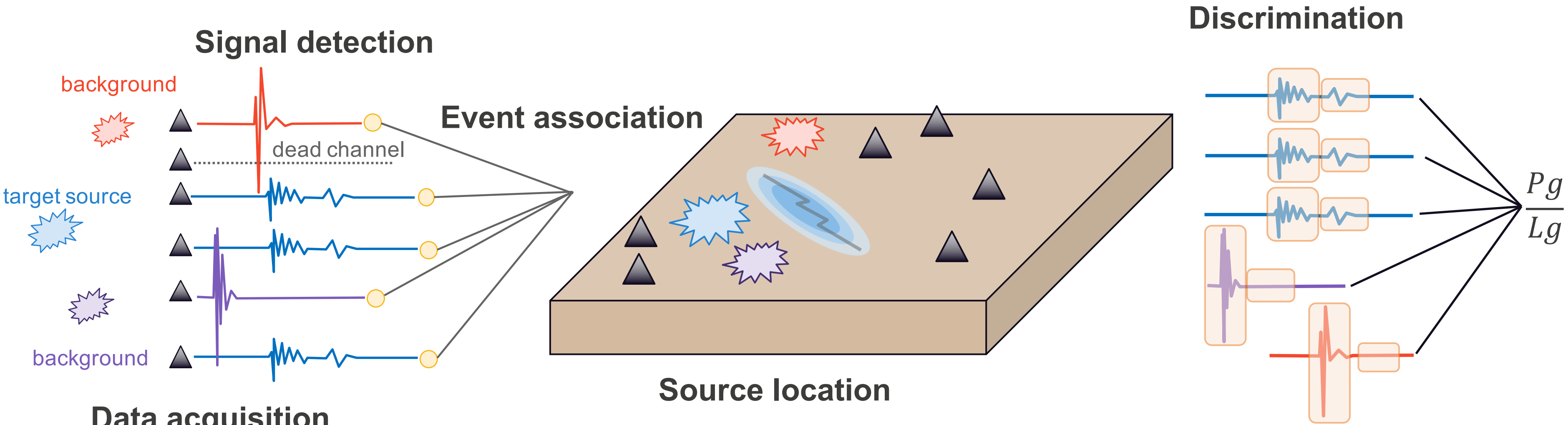
From Acquisition to Event Characterization

Problem: Quantify Gains in Event Processing Success (1/6)



The DNE23 Event Processing Pipeline proceeds as a set of analysis tasks to process and interpret signatures of target sources (like a LY-UNE). One goal of LYNM is to increase the probability that we can collectively identify and characterize this target source (with measurable error)

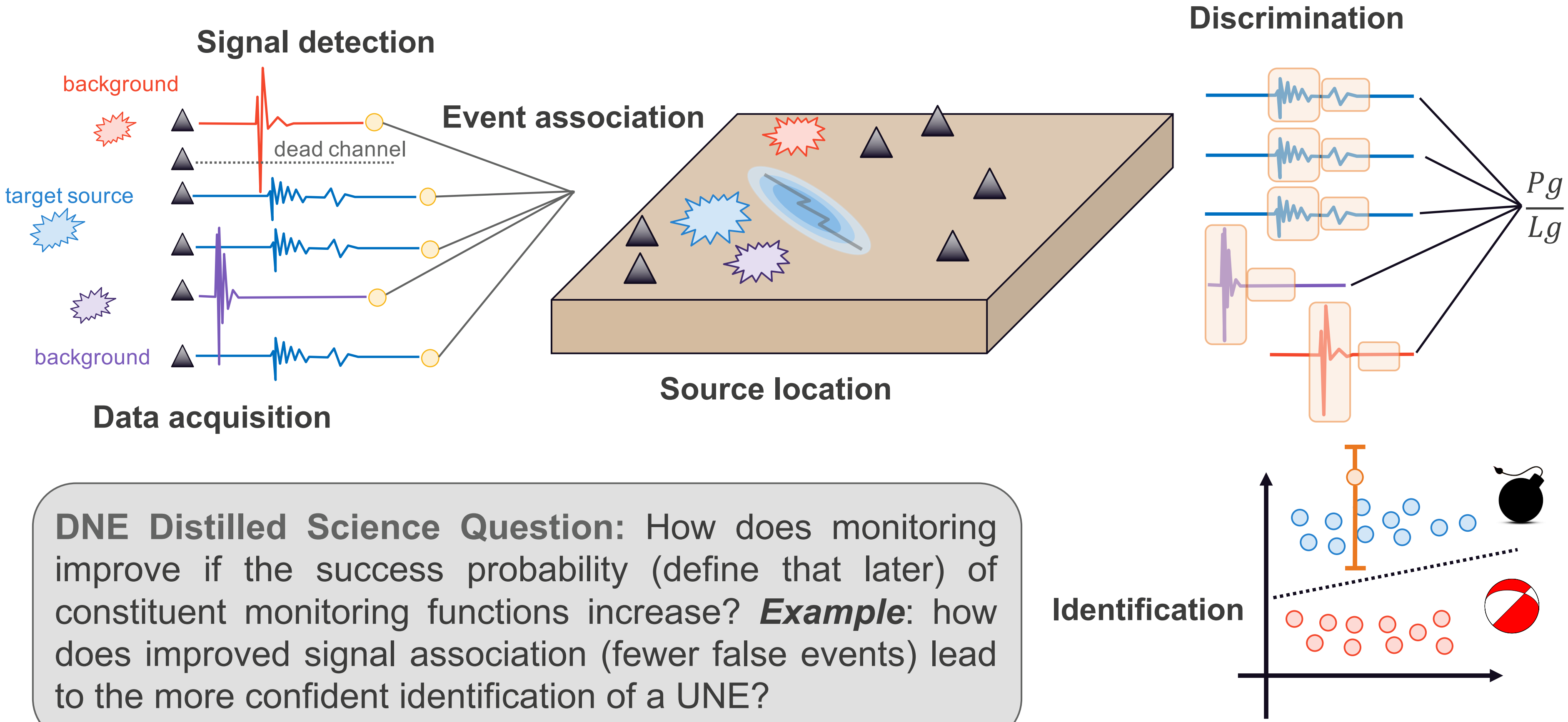
Problem: Quantify Gains in Event Processing Success (2/6)



The DNE23 Event Processing Pipeline proceeds as a set of analysis tasks to process and interpret signatures of target sources (like a LY-UNE). One goal of LYNM is to increase the probability that we can collectively identify and characterize this target source (with measurable error)

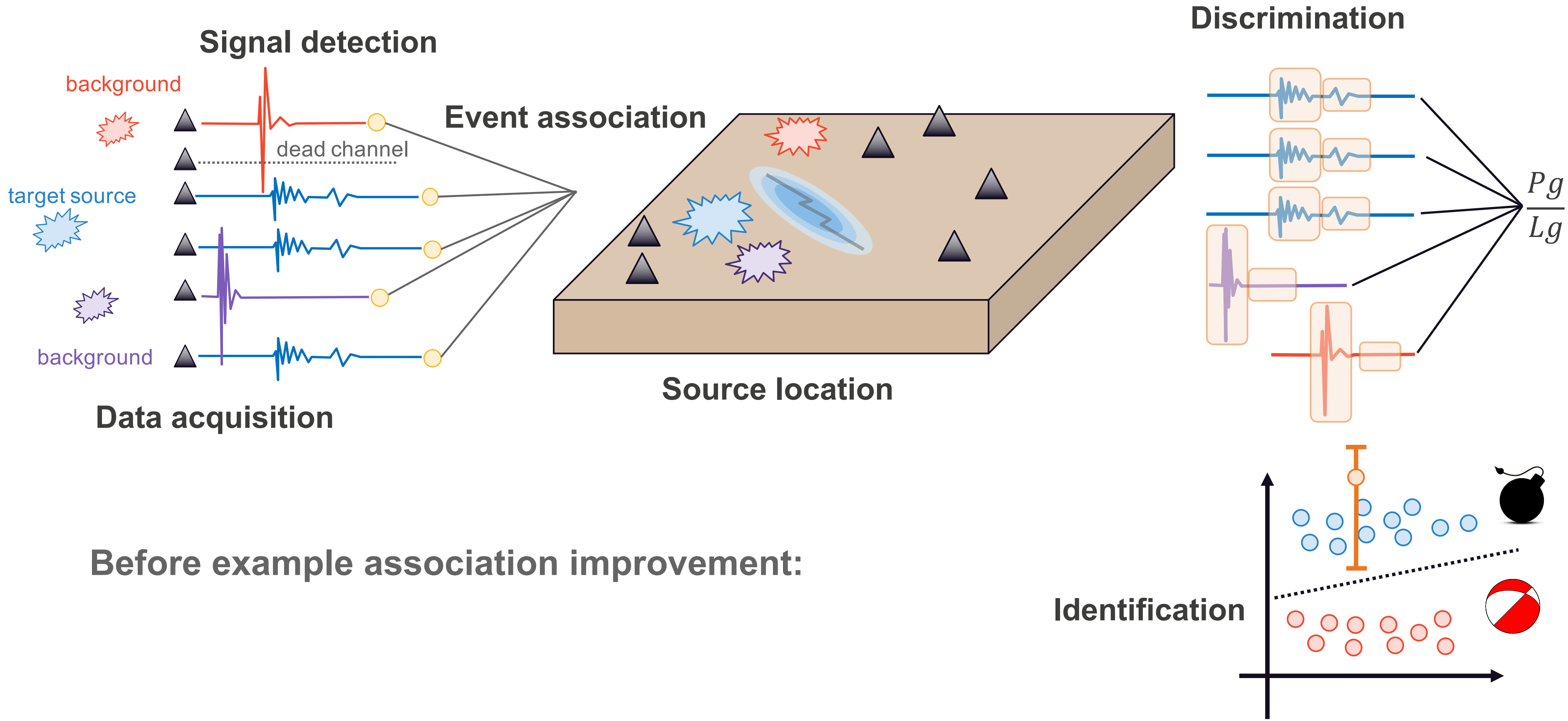
- Reduce false attribution rates, even with more data
- Reduce thresholds at which we can identify LY signatures
- Increase our predictive capability/ forecast what will detect and miss

Problem: Quantify Gains in Event Processing Success (3/6)



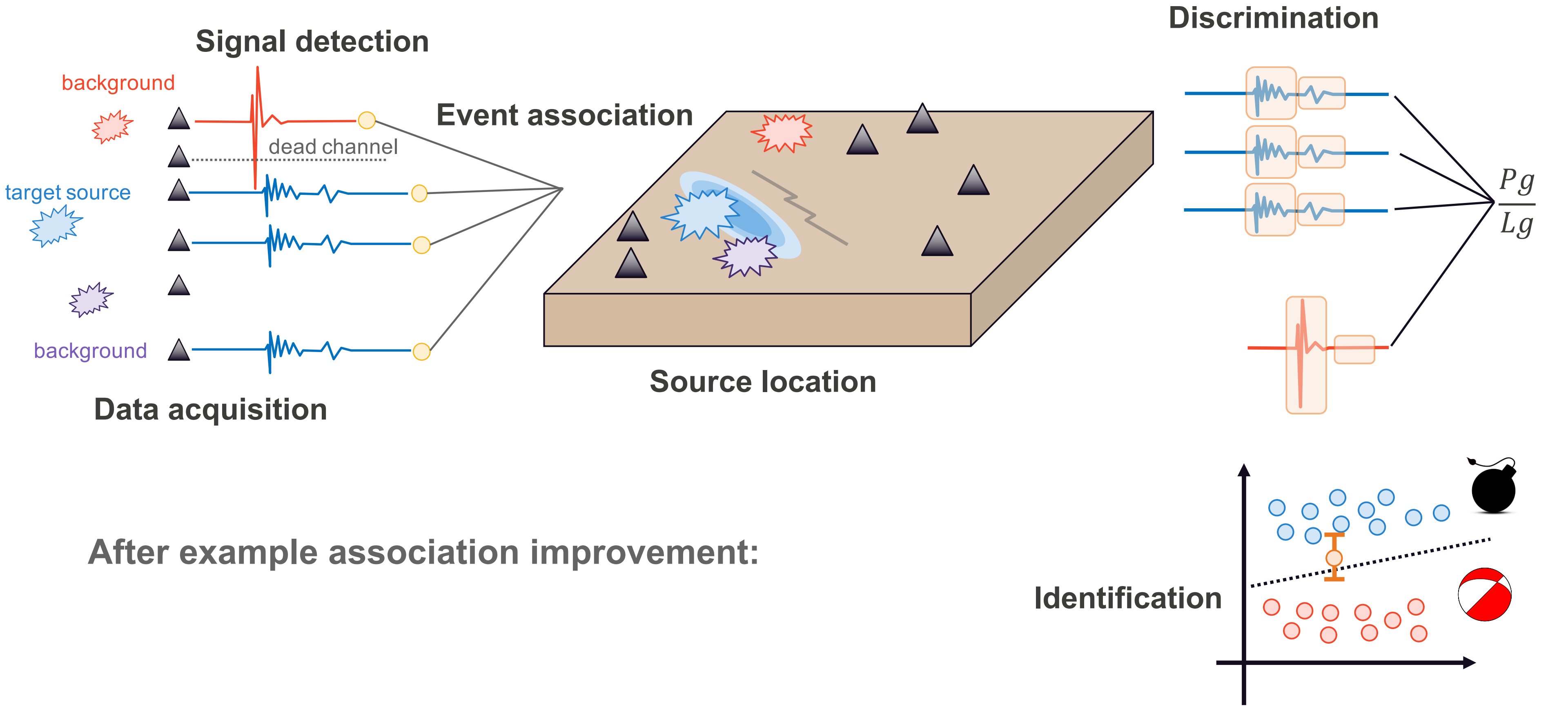
DNE Distilled Science Question: How does monitoring improve if the success probability (define that later) of constituent monitoring functions increase? ***Example***: how does improved signal association (fewer false events) lead to the more confident identification of a UNE?

Problem: Quantify Gains in Event Processing Success (4/6)



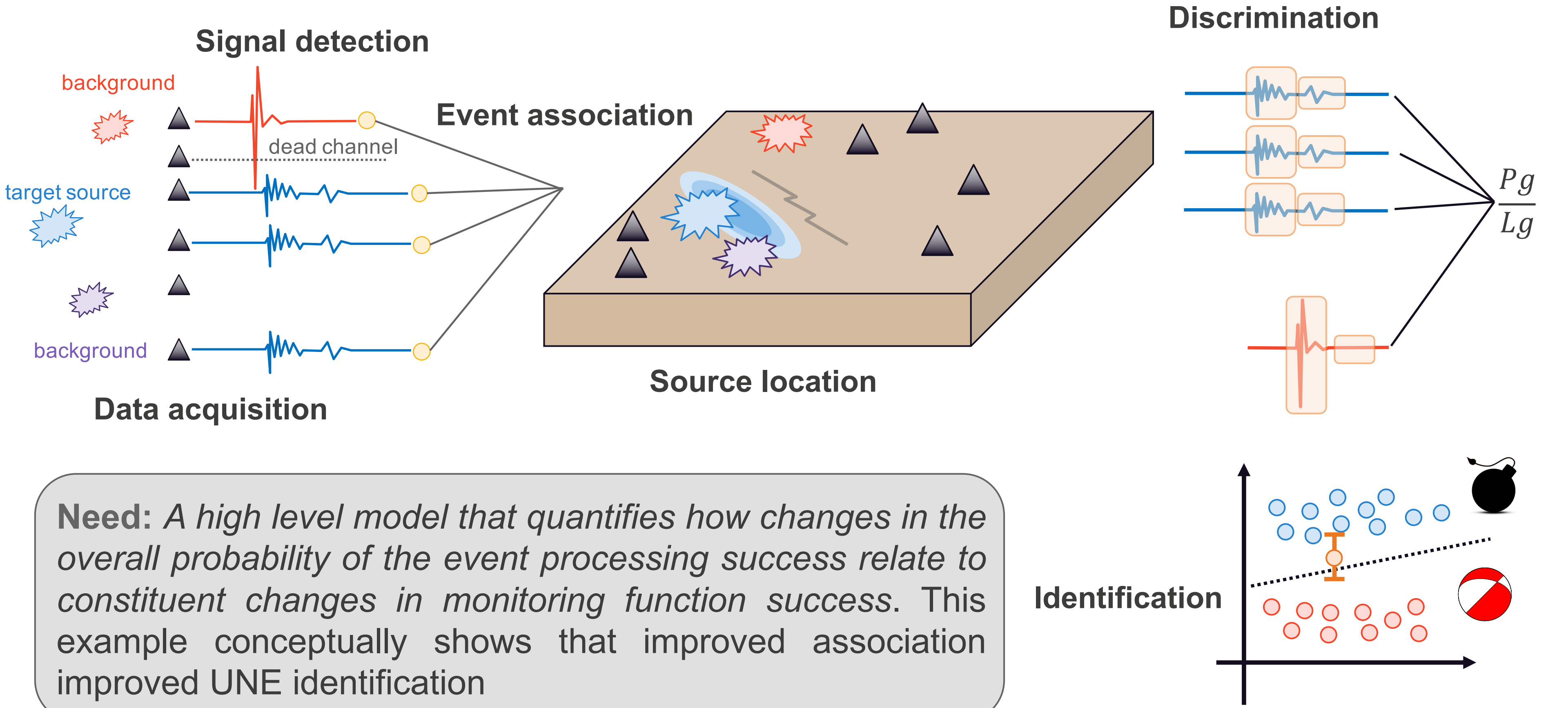
Before example association improvement:

Problem: Quantify Gains in Event Processing Success (5/6)



After example association improvement:

Problem: Quantify Gains in Event Processing Success (6/6)



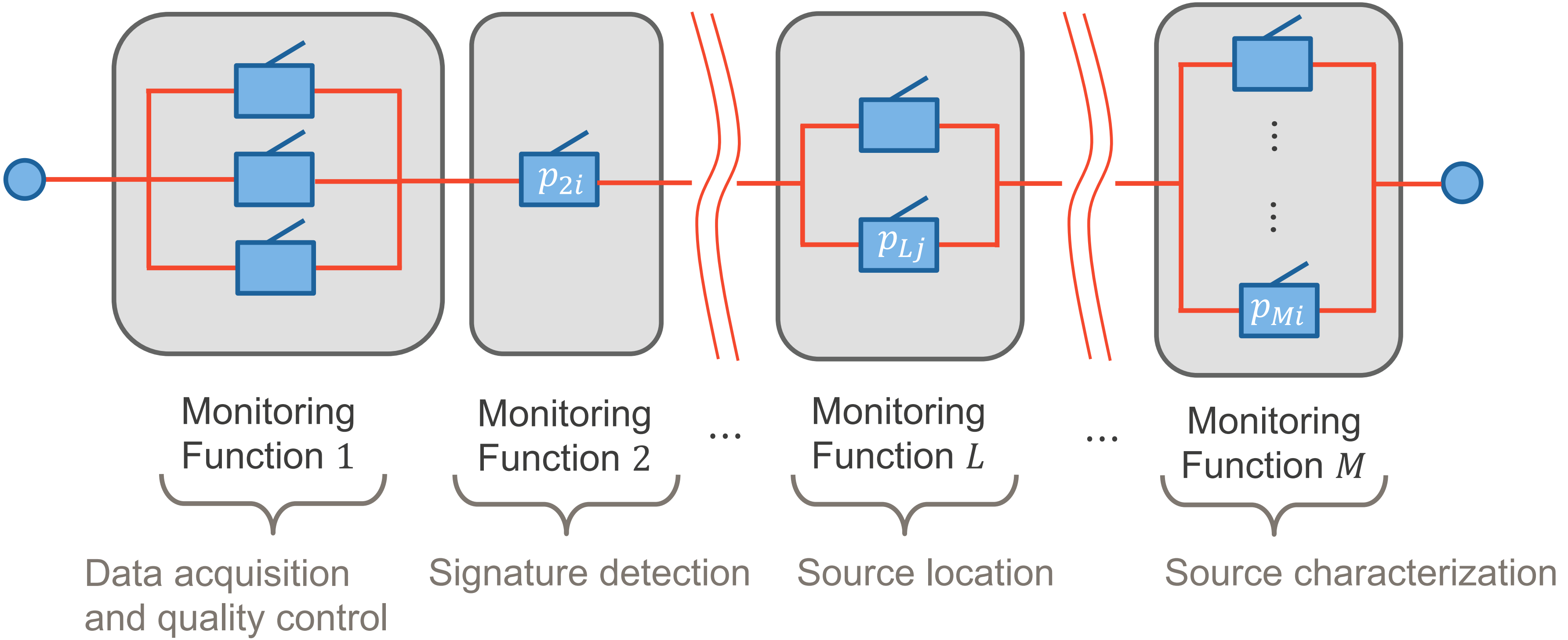
Need: *A high level model that quantifies how changes in the overall probability of the event processing success relate to constituent changes in monitoring function success.* This example conceptually shows that improved association improved UNE identification

Abstract Model for the Event Processing Pipeline

Reliability Block Diagrams model Acquisition to Event Characterization Efforts

An Event Processing Pipeline Reliability Model (1/3)

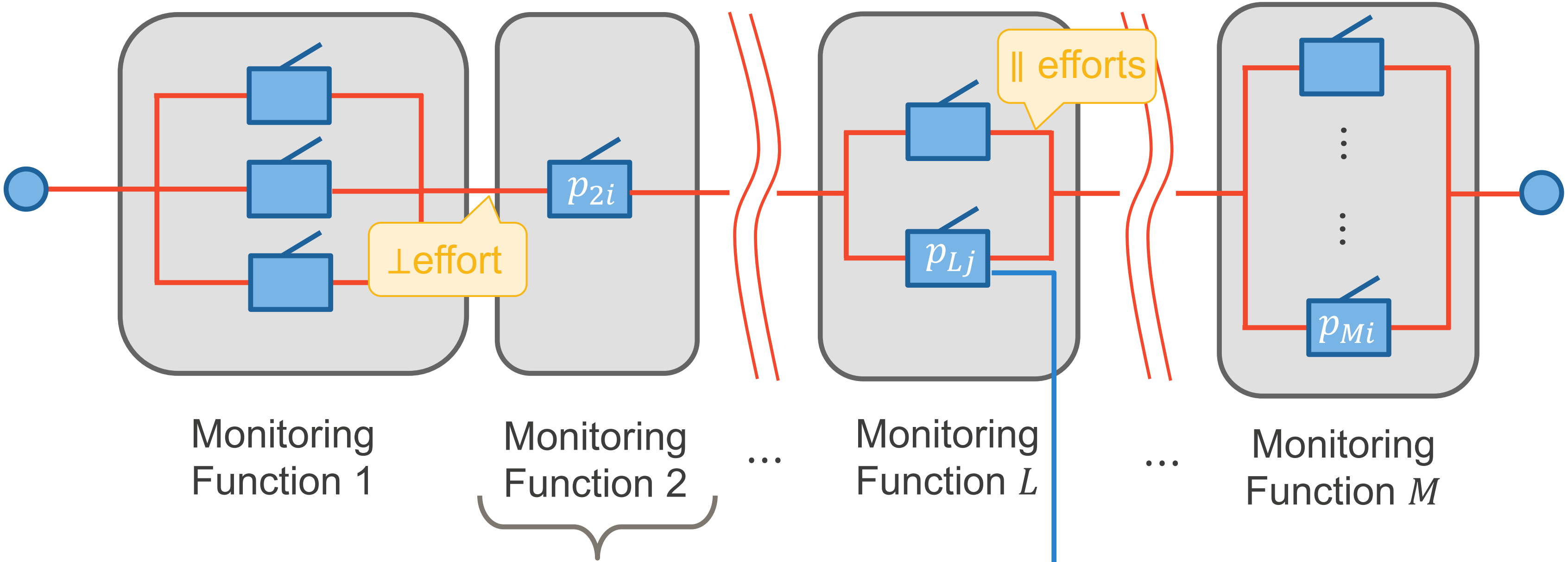
p_{kj} **Agent** j achieves monitoring function k with probability $1 - p_{kj}$, possibly in series or parallel with other agents



An Event Processing Pipeline Reliability Model (2/3)

Agents: $1, 2, \ldots, j \ldots, M$ perform sub-tasks for each monitoring function

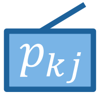
Probabilities: $1 - p_{jk}$ quantifies the rate at which agent j achieves monitoring function k



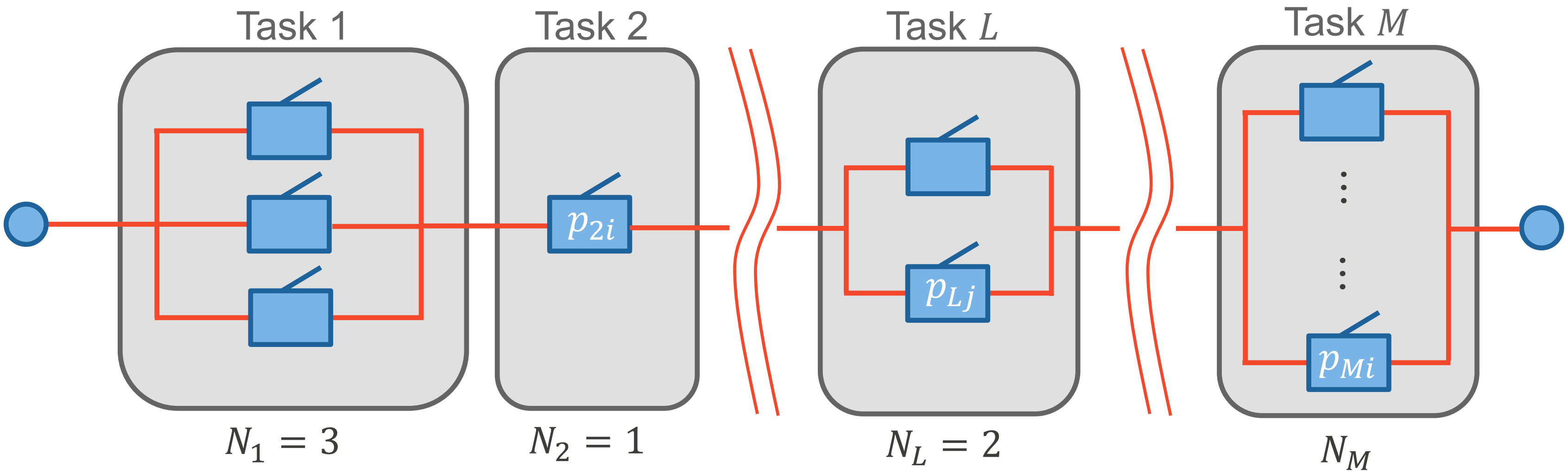
Agent i that completes Monitoring Function 2 with probability $p_{2i} < 1$ is a single point of failure

p_{Lj} = probability Agent j completes function L in parallel with another agent

An Event Processing Pipeline Reliability Model (3/3)



Agent j achieves $\boxed{\text{monitoring function } k}$ with probability $1 - p_{kj}$, possibly in series or parallel —— with other agents



$$\Pr_F(\text{EEP}) = 1 - \prod_{k=1}^{M}\left(1 - \prod_{l=1}^{N_k} p_{kl}^{\parallel}\right)$$

General form for the probability $\Pr_F(\text{EPP})$ that agents "fail" to complete the event processing pipeline to its final stage

Qualitative Ideas for Success/Failure Rate Reporting

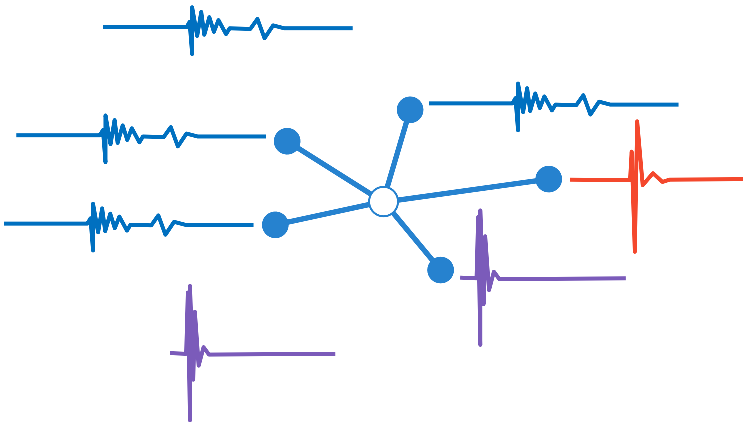
Association and Location Examples

$$\Pr_F(\mathrm{EPP}) = 1 - \prod_{k=1}^{M}\left(1 - \prod_{l=1}^{N_k} \boxed{p_{kl}^{\parallel}}\right)$$

Semi-Empirical Measurements of Agent Improvement (1/3)

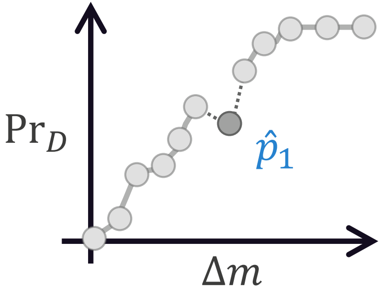
Association

t_1



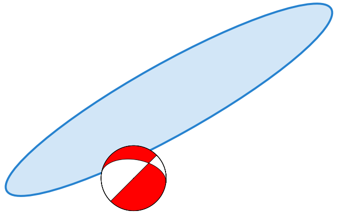
$$p = \frac{N_{\text{true}}}{N_{\text{Tot}}}$$

$$\hat{p} = \frac{\sum_{t=1}^{1} \frac{n_t}{n_T}}{\sum_{t=1}^{1} N_t}$$



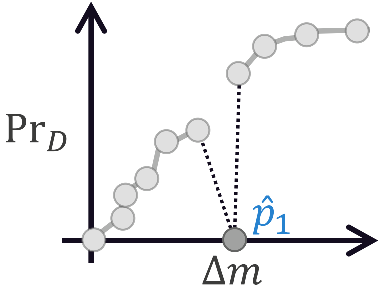
Source Location

t_1



$$p = \frac{N_{\text{in}}}{N_{\text{Tot}}}$$

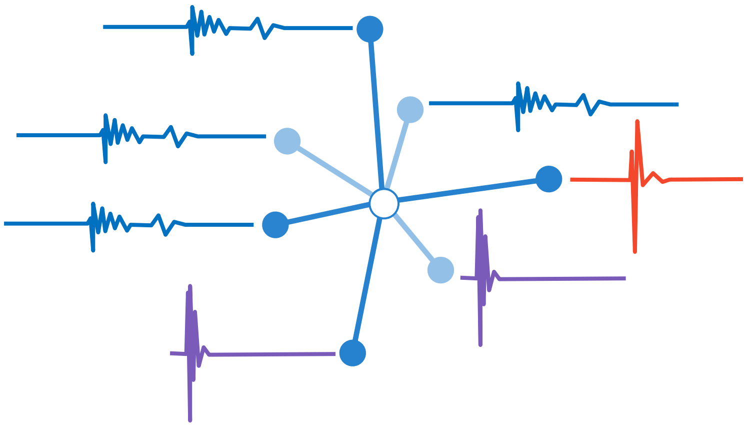
$$\hat{p} = \frac{\sum_{t=1}^{1} I_t}{N_T}$$



Semi-Empirical Measurements of Agent Improvement (2/3)

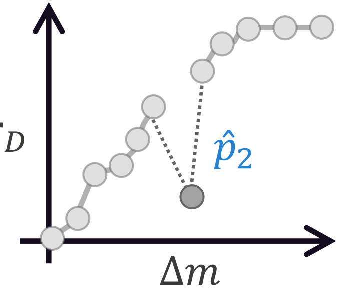
Association

t_1, t_2



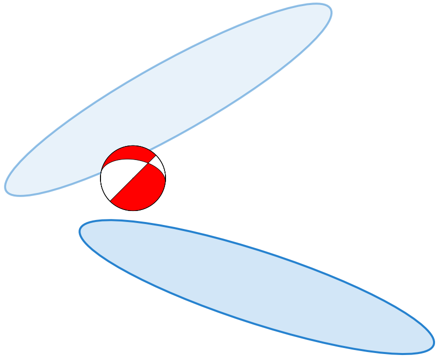
$$p = \frac{N_{\text{true}}}{N_{\text{Tot}}}$$

$$\hat{p} = \frac{\sum_{t=1}^{2} \frac{n_t}{n_T}}{\sum_{t=1}^{2} N_t}$$



Source Location

t_1, t_2



$$p = \frac{N_{\text{in}}}{N_{\text{Tot}}}$$

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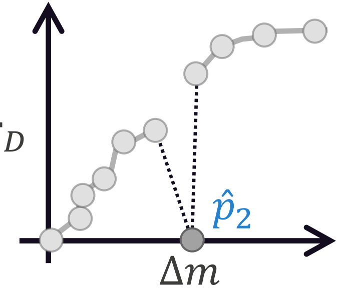
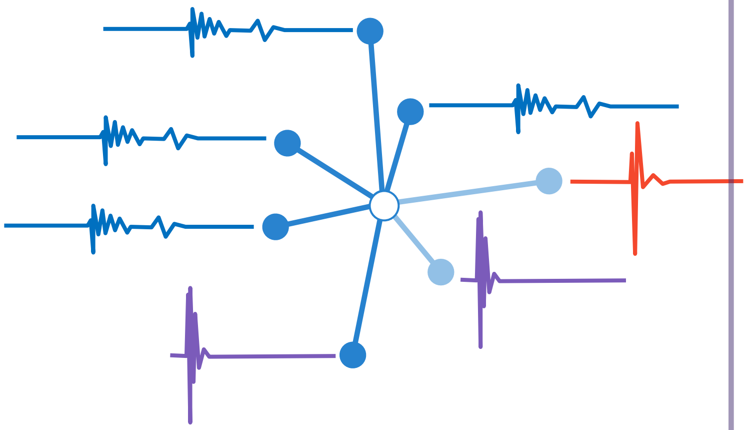


Semi-Empirical Measurements of Agent Improvement (3/3)

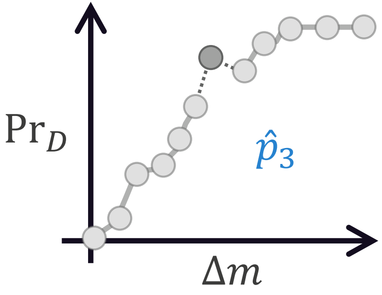
Association

t_1, t_2, t_3



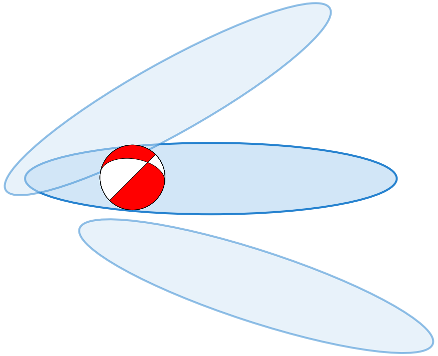
$$p = \frac{N_{\mathrm{true}}}{N_{\mathrm{Tot}}}$$

$$\hat{p} = \frac{\sum_{t=1}^{3} \frac{n_t}{n_T}}{\sum_{t=1}^{3} N_t}$$



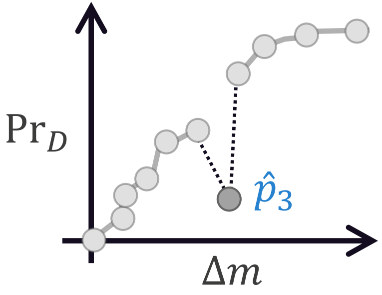
Source Location

t_1, t_2, t_3



$$p = \frac{N_{\mathrm{in}}}{N_{\mathrm{Tot}}}$$

$$\hat{p} = \frac{\sum_{t=1}^{3} I_t}{N_T}$$



Quantification of Gains and Losses for DNE23

Computing Focus Area Agnostic Success Probabilities

Problem Statement

Formal Problem Statement: We quantify the relative gain to the LYNM-DN event processing probability $\Pr_S(\mathrm{EPP})$ (system reliability) that results from improvement to constituent monitoring function performance:

$$\frac{\Delta\Pr_S(\mathrm{EPP})}{\Pr_S^{(0)}(\mathrm{EPP})} = 1 - \frac{\Pr_S^{(1)}(\mathrm{EPP})}{\Pr_S^{(0)}(\mathrm{EPP})}$$

$\Delta\Pr_S(\mathrm{EPP})$ is the change in EPP success probability, from a baseline success rate $\Pr_S^{(0)}(\mathrm{EPP})$.

$\Pr_S^{(1)}(\mathrm{EPP})$ is the probability of EPP success, after a gain in a single monitoring function's success probability.

Problem Statement; Types of Changes that Lead to EPP Change

Formal Problem Statement: We quantify the relative gain to the LYNM-DN event processing probability $\mathrm{Pr}_S(\mathrm{EPP})$ (system reliability) that results from improvement to constituent monitoring function performance:

$$\frac{\Delta \mathrm{Pr}_S(\mathrm{EPP})}{\mathrm{Pr}_S^{(0)}(\mathrm{EPP})} = 1 - \frac{\mathrm{Pr}_S^{(1)}(\mathrm{EPP})}{\mathrm{Pr}_S^{(0)}(\mathrm{EPP})}$$

1. Researchers and their algorithms (**agents**) each have their own reliability value (success probability)

2. **Workflow** efforts in series and in parallel can change achieve each monitoring function output

3. Resource limited teams use an SME as agents to work **multiple** monitoring functions (burdens)

Problem Solution: EPP Success Rates from a Single Agent

Agent Gain Measurably Improves EPP: SME i measures a probabilistic gain δp in their monitoring function algorithm "2" over its baseline p. The success rate of monitoring function "2" is $\Pr_S^{(0)}(C)$. The relative improvement is:

$$\frac{\Delta \Pr_S(\mathrm{EPP})}{\Pr_S^{(0)}(\mathrm{EPP})} = \frac{-\delta p_{2i}}{p_{2i}} \cdot \left(\frac{1 - \Pr_S^{(0)}(C_2)}{\Pr_S^{(0)}(C_2)} \right)$$

If multiple monitoring functions ***measurably*** improve, we measure the collective gain in monitoring capability as:

$$\Delta \Pr_S(\mathrm{EPP}) = \Pr_S^{(0)}(\mathrm{EPP}) \sqrt{\sum_{k=1}^{M} \left[\frac{-\delta p_k}{p_k} \cdot \left(\frac{1 - \Pr_S^{(0)}(C_k)}{\Pr_S^{(0)}(C_k)} \right) \right]^2}$$

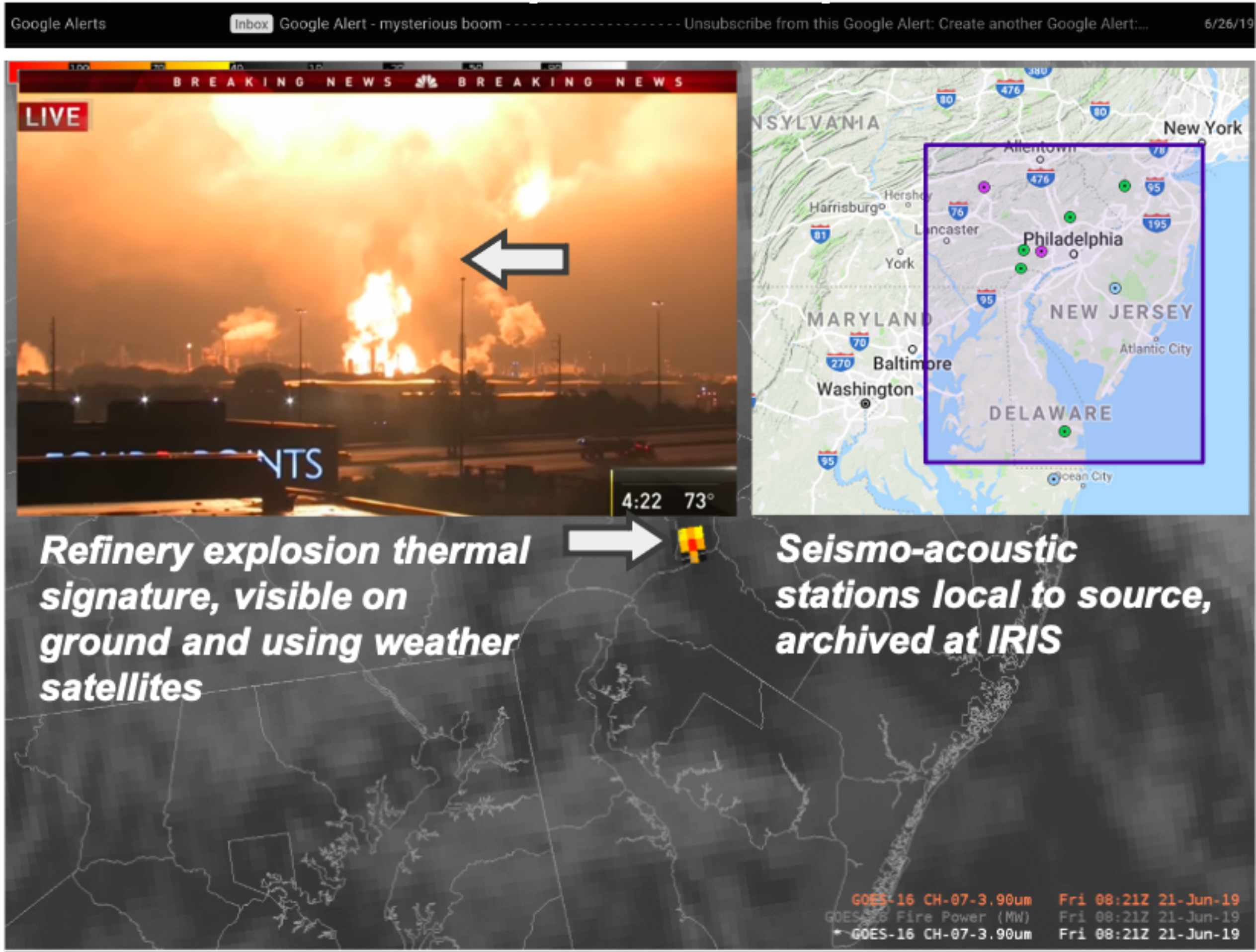
Example: Improvements to Signal Detection Measure Gain in EPP Success Rates

Improvements to a Correlation Detector Agent and EPP Improvement

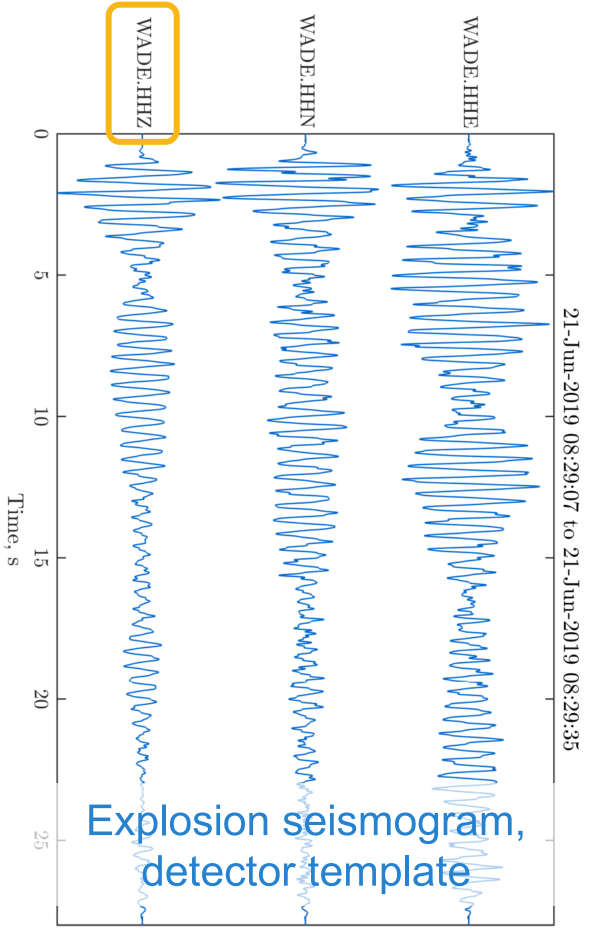
Example: A Near Surface, East Coast Explosion "Special Event"

Signal Detection Problem

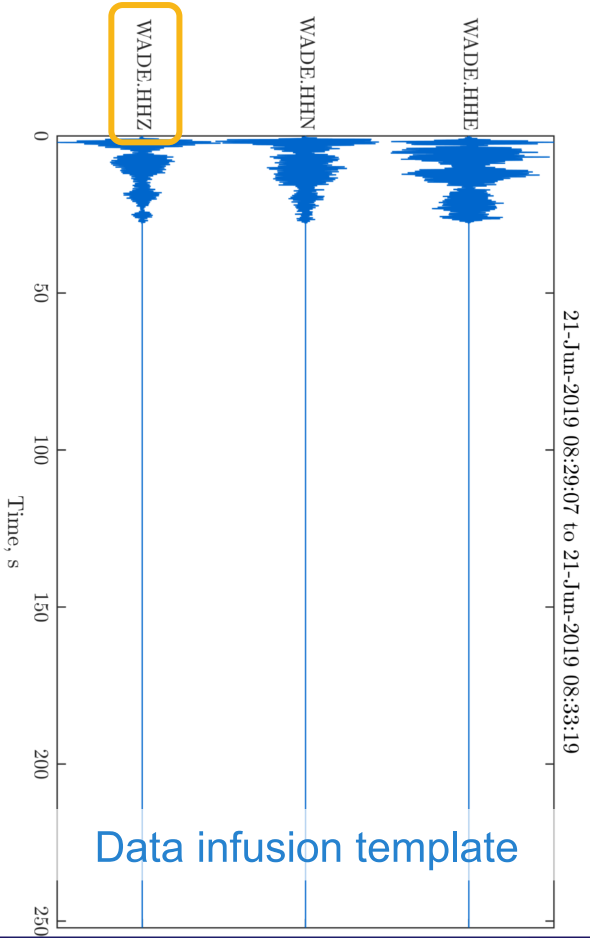
- Chemical explosion captured by multiple sensors that recorded multi-physics signatures
- No cataloged seismic event at USGS or IRIS, but my manual retrieval showed evident ground-coupled surface waves
- Will use exercise the signal detection monitoring function on waveforms that record this **special event**, with both a single channel correlation detector (**the baseline**), and then a three-channel correlation detector (**agent gain**)



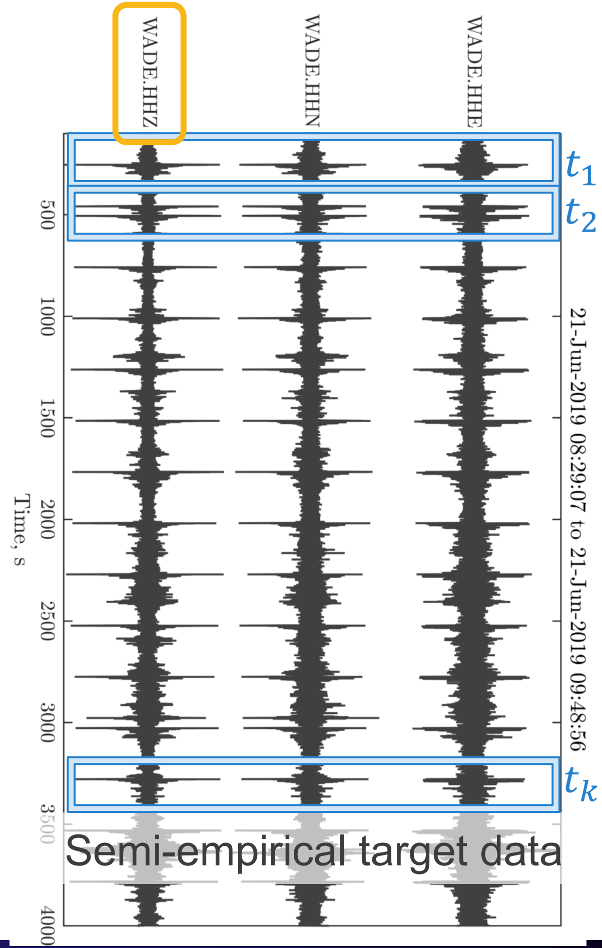
Semi-Empirical Performance of Baseline and Improved Detector



Explosion seismogram, detector template

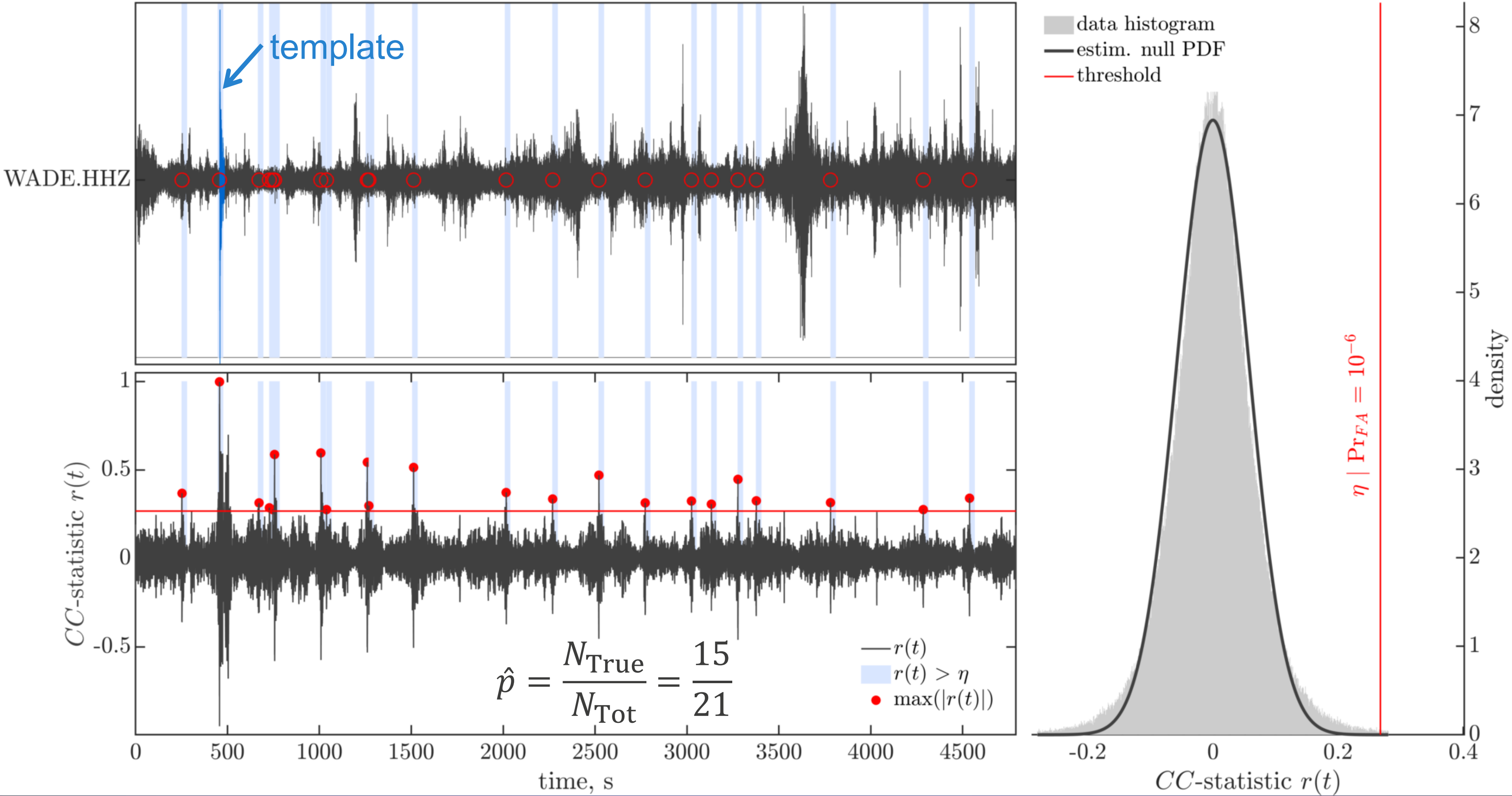


Data infusion template



Semi-empirical target data

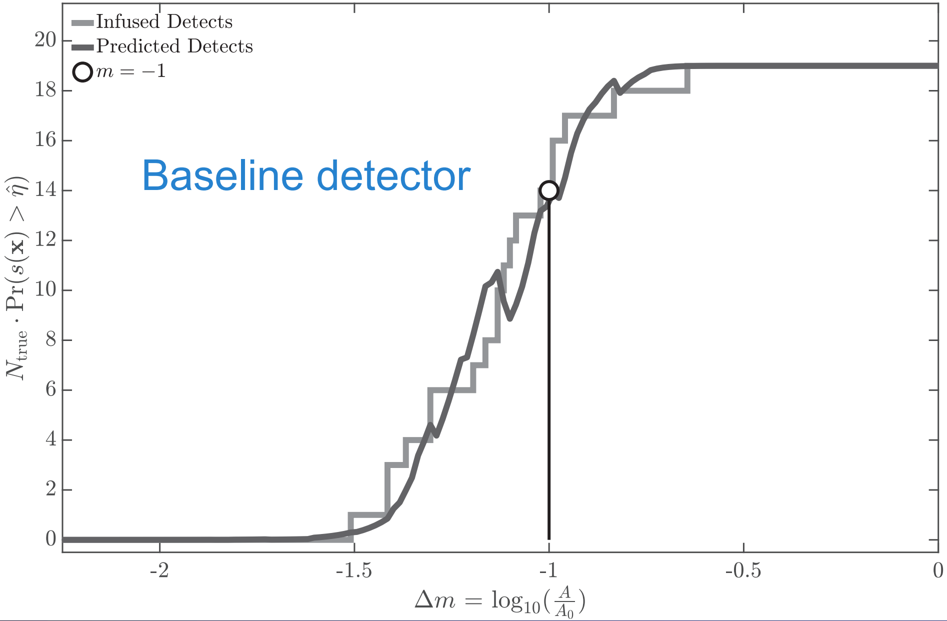
Step 1: Enumerate Detection Counts on Semi-Empirical Targets

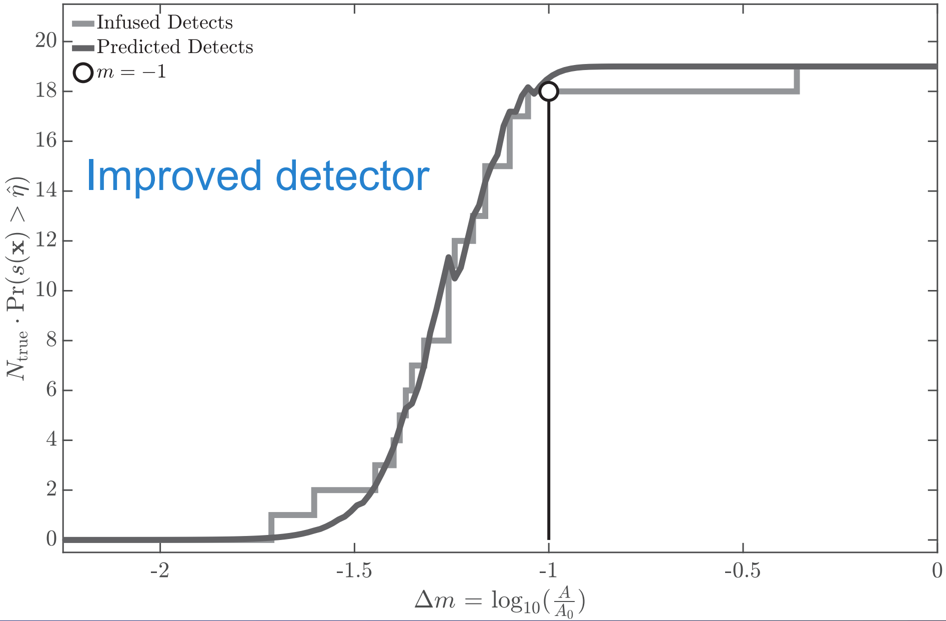


Step 2: Make Performance Curves to Quantify Detector "Success"

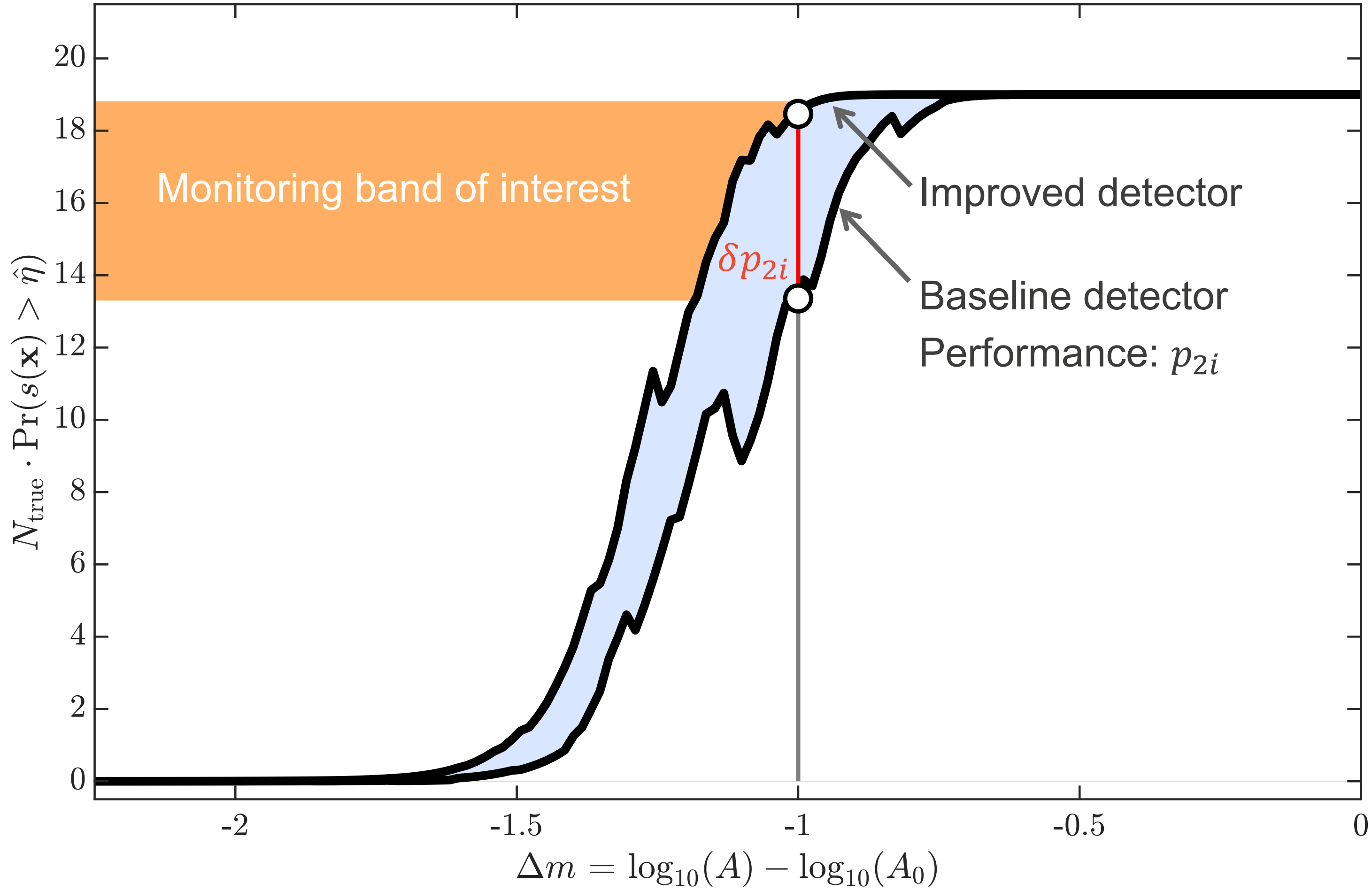
B
- Run single channel (HHZ) detector over an SNR / relative magnitude grid
- Ratio of true detection counts to total event count quantifies performance curve
- Theory-count agreement quantifies *predictive success*

G
- Run three channel (ENZ) detector over an SNR / relative magnitude grid
- Ratio of true detection counts to total event count quantifies performance curve
- Theory-count agreement quantifies *predictive success*; detector more sensitive

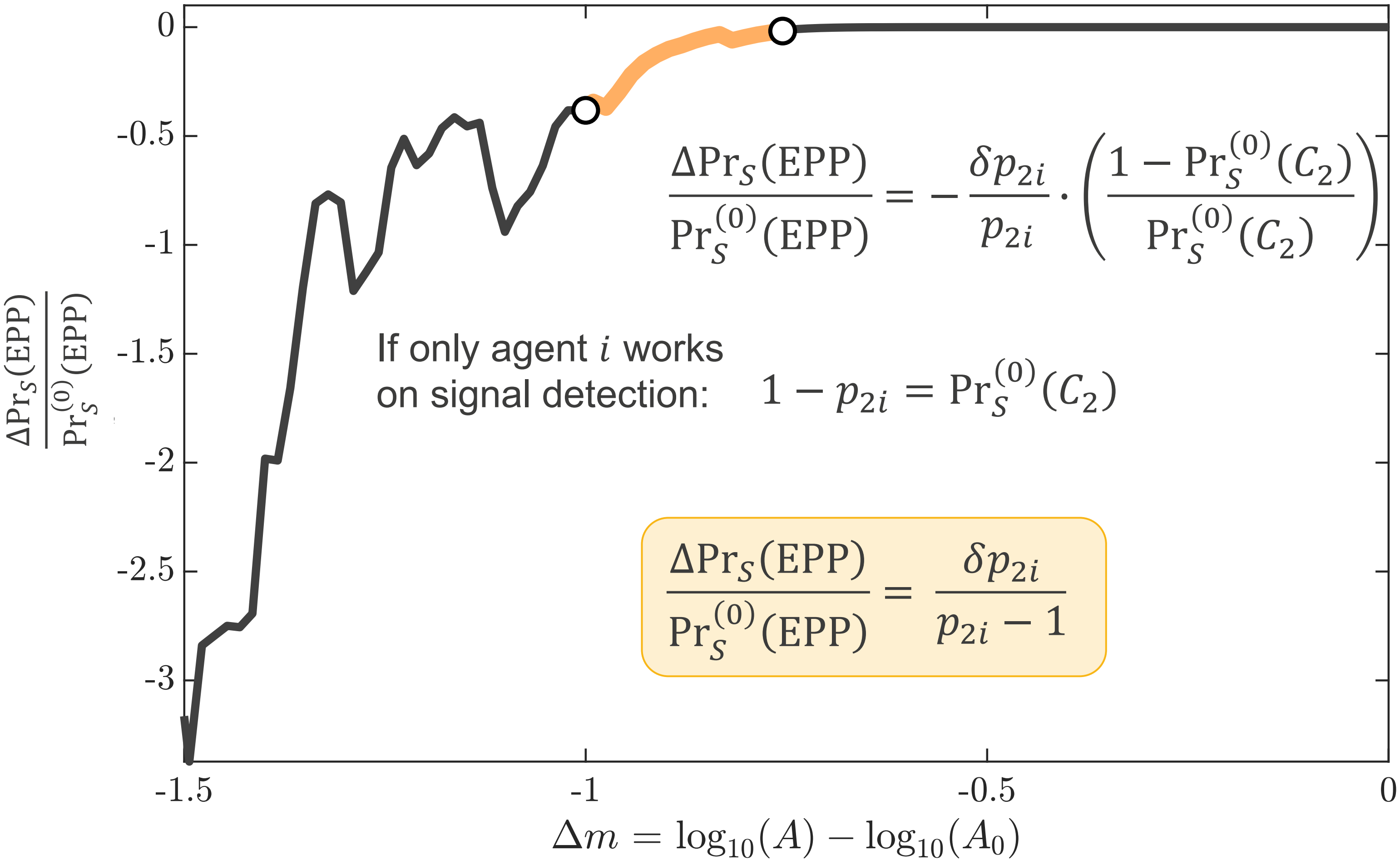




Step 2: Estimate Incremental Gain in Success Probability



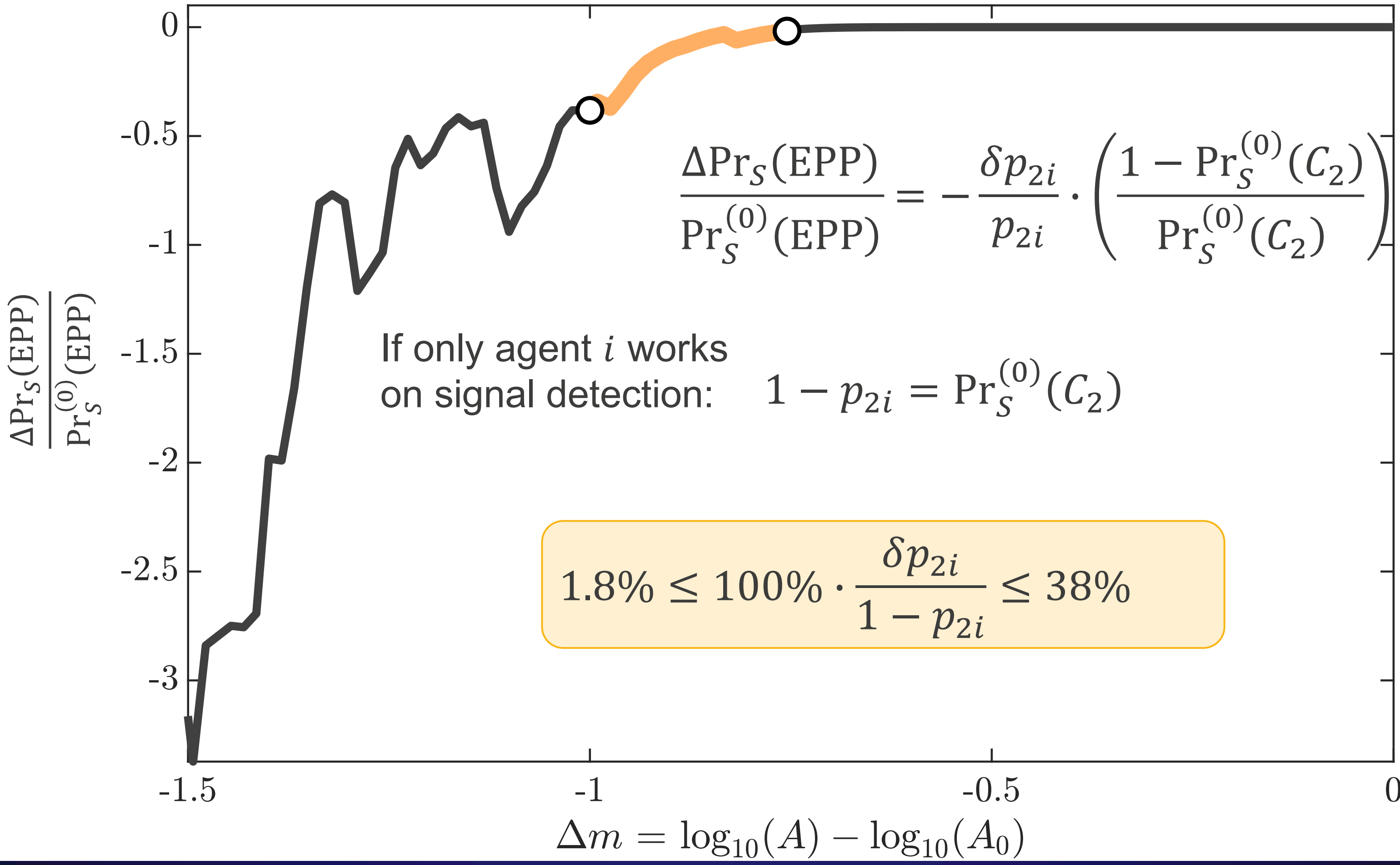
Step 3: Compute Relative Detector Improvement, $\delta p/p$ (1/2)



$$\frac{\Delta \mathrm{Pr}_S(\mathrm{EPP})}{\mathrm{Pr}_S^{(0)}(\mathrm{EPP})} = -\frac{\delta p_{2i}}{p_{2i}} \cdot \left(\frac{1 - \mathrm{Pr}_S^{(0)}(C_2)}{\mathrm{Pr}_S^{(0)}(C_2)} \right)$$

If only agent i works on signal detection: $1 - p_{2i} = \mathrm{Pr}_S^{(0)}(C_2)$

$$\boxed{\frac{\Delta \mathrm{Pr}_S(\mathrm{EPP})}{\mathrm{Pr}_S^{(0)}(\mathrm{EPP})} = \frac{\delta p_{2i}}{p_{2i} - 1}}$$



$$\frac{\Delta \mathrm{Pr}_S(\mathrm{EPP})}{\mathrm{Pr}_S^{(0)}(\mathrm{EPP})} = -\frac{\delta p_{2i}}{p_{2i}} \cdot \left(\frac{1-\mathrm{Pr}_S^{(0)}(C_2)}{\mathrm{Pr}_S^{(0)}(C_2)} \right)$$

If only agent i works on signal detection: $1 - p_{2i} = \mathrm{Pr}_S^{(0)}(C_2)$

$$1.8\% \leq 100\% \cdot \frac{\delta p_{2i}}{1-p_{2i}} \leq 38\%$$

Appendices (A-D)

A. Vocabulary of Event Processing Pipeline Components
B. Reliability Block Diagrams
C. Graph theory Relations to Enable Computation
D. Optimal EPP Agent Network Topologies (Graphs)

A: Vocabulary of Event Processing Pipeline Components

Formal Definitions

Elements of Event Processing Pipeline (1/3)

Monitoring functions are a *series* of analysis tasks $(1,2,\ldots,k,\ldots,M\ )$ that output parametric estimates of geophysical signatures and their sources, such that each task increases the cumulative knowledge of that source. *Example*: data acquisition, signal detection, association/event building, source location, discrimination, and identification are each (but perhaps not all) monitoring functions required to partially characterize an underground explosion source.

Agents are any number $1,2,\ldots,j,\ldots,N$ of algorithms and/or subject matter experts (SMEs) that attempt monitoring function tasks. *Example*: a team of 10 researchers with 12 data processing algorithms collectively represent $N = 12$ agents. Agent 1 and j are each SMEs of the discrimination monitoring function.

Burdened agents refer to those agents in an EPP that attempt more than a single monitoring function to analyze the same special event. *Example*: An SME that applies algorithms to both signal detection and source location to data collected from the same underground explosion is a burdened agent.

Elements of Event Processing Pipeline (2/3)

Failure of Agent j at monitoring function k means that their data product or decision output includes sufficiently large errors such that following monitoring functions can only output errors that exceed some admissible bound. *Example*: Agent j links infrasound waveforms that arrive at one array, which are sourced by an explosion, with infrasound waveforms that arrive at a second array, which are sourced by bolide. Agent j then *fails* to achieve the association monitoring function.

Workflow topology: if failure of monitoring function $k-1$ causes failure of monitoring functions $\geq k$, then functions $k-1$ and k are in *series* (symbol $\perp$). Monitoring function k is completed *in parallel* (symbol ∥) by multiple agents if each agent can independently complete their task with nonzero probability, given input from function $k-1$ and requisite data. *Example*: a “single-point-of-failure" occurs if failure of Agent j to perform association within a team of M agents that work in series necessarily causes EPP failure.

Elements of Event Processing Pipeline (3/3)

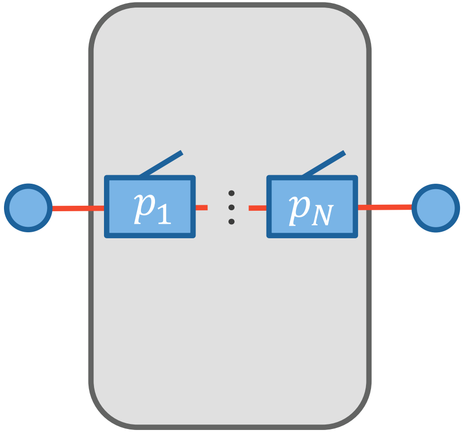
Workflow probability p_{kj} quantifies the rate that Agent j independently fails to achieve monitoring function k, given requisite data. Probability $1 - p_{jk}$ quantifies Agent j's probability of achieving monitoring function k. *Example*: Agent j processes waveform data of a located seismic source to compute a bandlimited, seismic phase ratio Pg/Lg discriminant with success rate $1 - p_{Lj}$, in which discrimination is monitoring function L.

B: Reliability Block Diagrams

Quantification of Parallel and Series Workflow Efforts

Probability of Success of Agents in Series

A series topology



Monitoring Function L

Agent j fails to complete Monitoring Function L with probability $p_{Lj}^{\perp}$

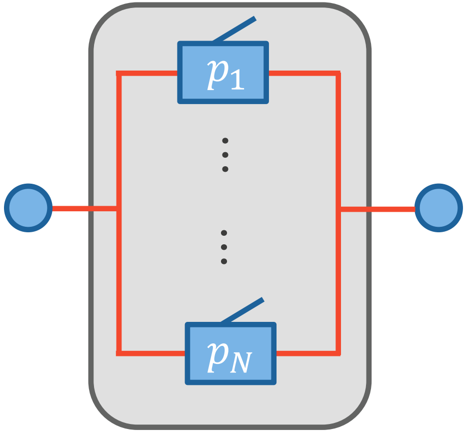
Monitoring function L is strictly a failure or a success: $\Pr(\bar{L}) + \Pr(L) = 1$

Probability that monitoring function L succeeds with N_L agents working in series:

$$\Pr(L) = \prod_{l=1}^{N_L} \left(1 - p_{Ll}^{\perp}\right)$$

Probability of Success of Agents in Parallel

A parallel topology



Monitoring Function L

Agent j fails to complete Monitoring Function L with probability $p_j^{\parallel}$

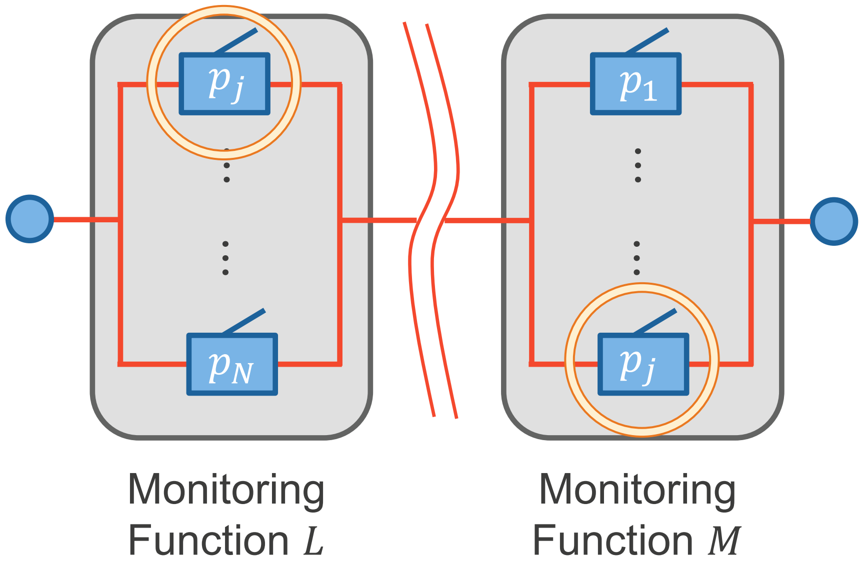
Monitoring function L is strictly a failure or a success: $\Pr(\bar{L}) + \Pr(L) = 1$

Probability that monitoring function L succeeds with N_L agents working in parallel:

$$\Pr(L) = 1 - \prod_{l=1}^{N_L} p_{Ll}^{\parallel}$$

Steady-State Probability of Failure for a "Burdened" Agent

If Agent j's task load is $N_j > 1$ monitoring functions, failure probability $p_j^{\parallel}$ increases by amount $N_j \varepsilon_j$, so that $p_j^{\parallel} = p_j^{\perp} + (N_j - 1)\varepsilon_j^{\parallel}$



Constraints

At least one agent works on each monitoring function (there are no "trivial" signal point failures)

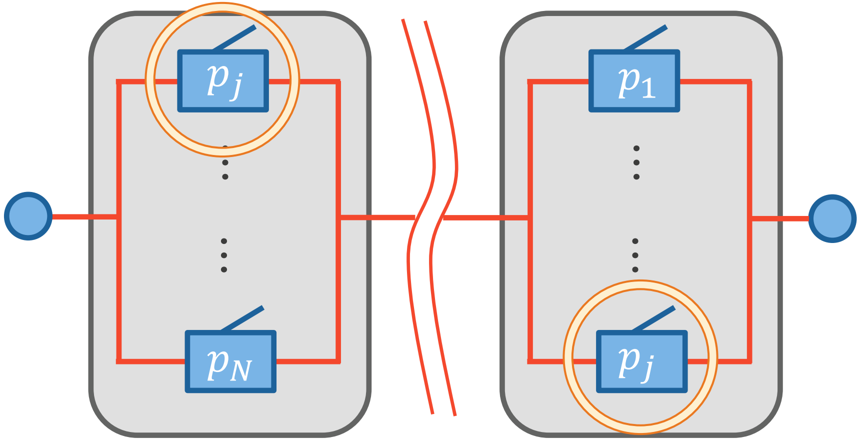
No agent has a task load that exceeds the number of monitoring functions M

Burdened agents' failure probability cannot exceed one: $\varepsilon_j^{\parallel} \leq p_j^{\perp}/(M-1)$

In practice: the failure probability of Agent j is the product that their task succeeds, multiplied by a (Bernoulli, for example) probability that their task can be completed

“Dynamic” Failure of a “Burdened” Agent (1/3)

Failure probability $p_j^{\parallel}$ must be 1 before characteristic completion time $\lambda\tau$ for a particular monitoring function. Then: $p_j^{\parallel} = p_j^{\perp}\left(1 + \left(N_j - 1\right)\varepsilon_j^{\parallel}(0)e^{-\lambda(t-\tau)}\right)$ for $t > \tau$



Monitoring Function L

Monitoring Function M

Constraints

The failure rate due to task splitting by a single agent should go to zero as time increases (note $\varepsilon_j^{\parallel}(0)$)

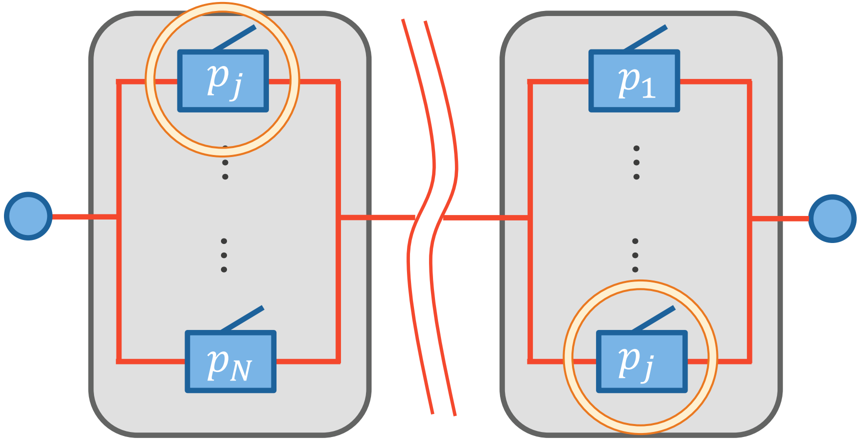
Parallel task failure rate should be one (1) for times less than $\lambda\tau$

Burdened agents’ failure probability still cannot exceed one: $\varepsilon_j^{\parallel} \leq p_j^{\perp}/(M-1)$

We bound EPP failure rate of a particular monitoring function by assuming the upper bound on failure rate is achieved at time $t = \tau$

"Dynamic" Failure of a "Burdened" Agent (2/3)

Scaling arguments on burden can define the exponential decay term λ. The dynamic failure probability for $t > \tau$: $p_j^{\parallel} = p_j^{\perp}\left(1 + (1 - p_j^{\perp})e^{-1 \cdot \frac{\ln(N_j)}{N_j - 1} \cdot (t-\tau)}\right)$ exponentially scales with task load



Monitoring Function L

Monitoring Function M

Constraints

The failure rate due to task splitting by a single agent should go to zero as time increases (note $\varepsilon_j^{\parallel}(0)$)

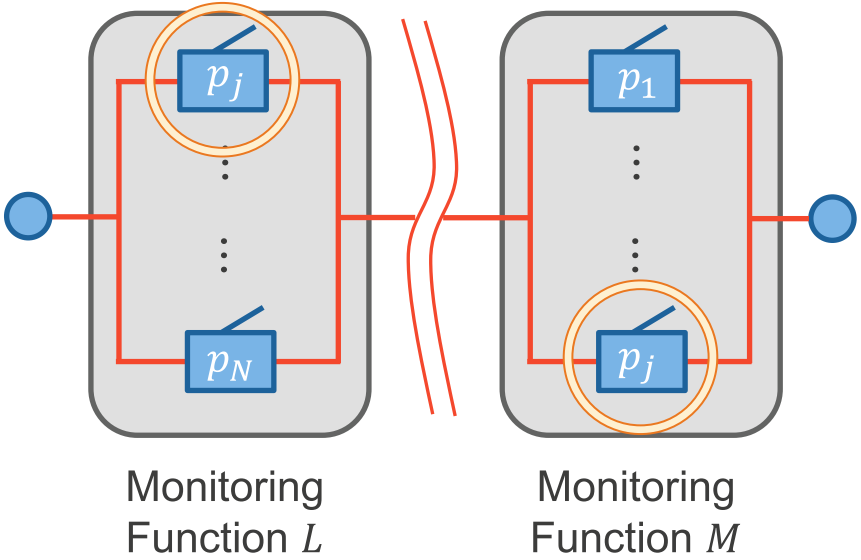
Parallel task failure rate should be one (1) for times less than $\lambda\tau$

Burdened agents' failure probability still cannot exceed one: $\varepsilon_j^{\parallel} \leq p_j^{\perp}/(M-1)$

We bound EPP failure rate of a particular monitoring function by assuming the upper bound on failure rate is achieved at time $t = \tau$

“Dynamic” Failure of a “Burdened” Agent (3/3)

Scaling arguments on burden can define the exponential decay term λ. The dynamic failure probability for $t > \tau$: $p_j^{\parallel} = p_j^{\perp}\left(1 + (1 - p_j^{\perp})e^{-1\cdot\frac{\ln(N_j)}{N_j - 1}\cdot(t-\tau)}\right)$ exponentially scales with task load



Constraints

The failure rate due to task splitting by a single agent should go to zero as time increases (note $\varepsilon_j^{\parallel}(0)$)

Parallel task failure rate should be one (1) for times less than $\lambda\tau$

Burdened agents’ failure probability still cannot exceed one: $\varepsilon_j^{\parallel} \leq p_j^{\perp}/(M-1)$

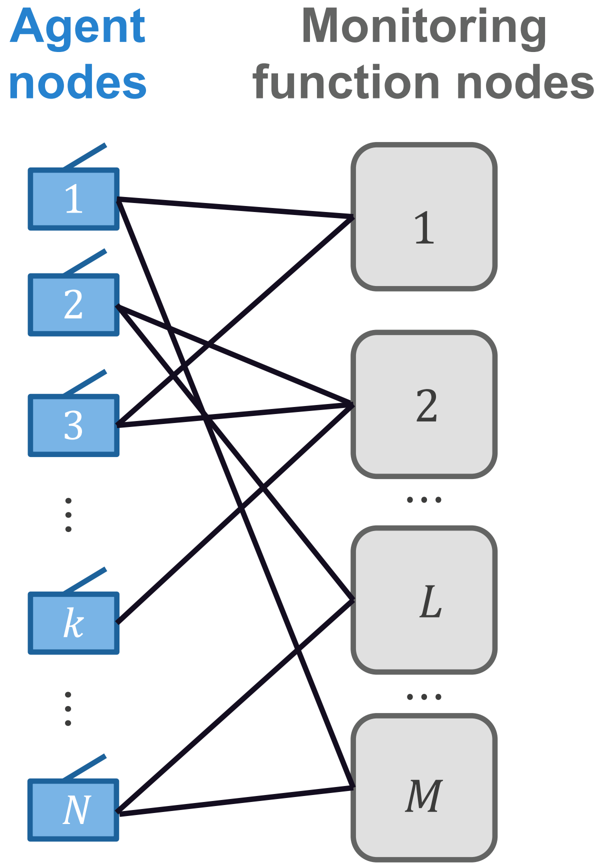
Agent and failure dynamics with coupled agents and feedbacks that quantify longer time-scale agent improvements comprise continuing work; remaining appendix excludes this content

C: Graph Theory Relations to Enable Computation

Bipartite Graphs to Link Agents to Monitoring Functions

Agent and Monitoring Function Graphical Relations

Bigraph



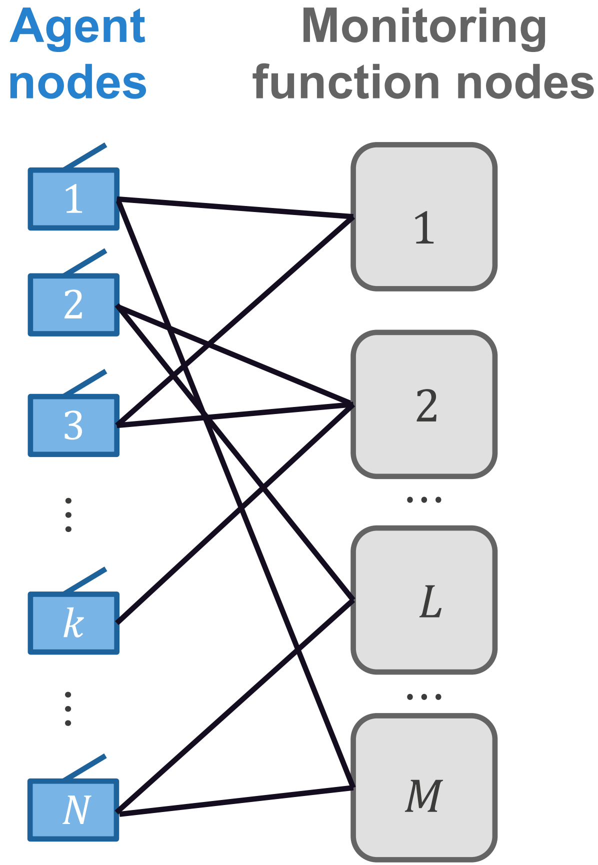
"Adjacency matrices" can represent graphs; this matrices have 1's to indicate if nodes connect by edges, which can be weighted to indicate the connection significance.

Here, agents represent certain nodes and monitoring functions represent other nodes. Agents connect to monitoring function nodes only, and the adjacency matrix $\boldsymbol{A}$ has a general form:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \boldsymbol{B} \\ \boldsymbol{B}^{\mathrm{T}} & \mathbf{0} \end{bmatrix}$$

Agent and Monitoring Function Graphical Relations

Bigraph



The bipartite matrix B in the graph adjacency matrix:

Monitoring function 2

Agent 2

$$B = \underbrace{\begin{matrix} 1 & 0 & \cdots & 0 & \cdots & 1 \\ 0 & 1 & \cdots & 1 & \cdots & 0 \\ 1 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 1 \end{matrix}}_{M \text{ columns index monitoring functions}} \left.\vphantom{\begin{matrix} 1 \\ 0 \\ 1 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{matrix}}\right\} N \text{ rows index agents}$$

Compute $\Pr_F(\text{EPP})$ Parameters from Graph Adjacency Matrix

Number of monitoring functions (MFs):

$$M = \text{size}(\boldsymbol{B}, 2)$$

Number of ‖ efforts directed at MF k:

$$N_k = \mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_k$$

Burden number for Agent j:

$$N_j = \hat{\boldsymbol{e}}_j^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \mathbf{1}$$

Elementary vectors ($M \times 1$ or $N \times 1$):

$$\hat{\boldsymbol{e}}_j^{\mathrm{T}} = [0 \quad \ldots \quad \underbrace{1}_{\text{column } j} \ldots \quad 0]$$

$$\mathbf{1}^{\mathrm{T}} = [1 \quad \ldots \quad 1 \quad \ldots \quad 1]$$

$$B_{jk} = \begin{cases} 1, & \text{Agent } j \text{ works MF } k \\ 0, & \text{otherwise} \end{cases}$$

$$\underbrace{\left.\begin{matrix} 1 & 0 & \cdots & 0 & \cdots & 1 \\ 0 & 1 & \cdots & 1 & \cdots & 0 \\ 1 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 1 \end{matrix}\right\}}_{M \text{ columns index monitoring functions}} N \text{ rows index agents}$$

Defining Failure Probabilities for Agents (1)

A team of SMEs includes six agents must complete seven monitoring functions to complete analyses of a special event located in a tectonic region.

\# *Monitoring function (number of capable agents)*

1. Data acquisition and quality control ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_1 = 3$)
2. Signature detection ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_2 = 2$)
3. Association and event building ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_3 = 2$)
4. Phase picking and location ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_4 = 3$)
5. Discrimination and screening ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_5 = 1$)
6. Source identification ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_6 = 1$)
7. Source characterization ($\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_7 = 2$)

$$
\begin{array}{c|ccccccc}
 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
4 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
 & & & & & & & 7
\end{array}
$$

Highlighted example:

Agent 4 is an SME for data acquisition, waveform association, and location

Source characterization (Monitoring Function 7) can be performed by two SMEs

Computational System to Estimate EPP Failure Rate (1/3)

$$\Pr_F(\text{EPP}) = 1 - \prod_{k=1}^{\text{size}(\boldsymbol{B},2)} \left(1 - \prod_{j=1}^{\mathbf{1}^{\mathrm{T}}\cdot\boldsymbol{B}\cdot\hat{\boldsymbol{e}}_k} \left[p_{kj}^{\perp} + \left(\hat{\boldsymbol{e}}_j^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \mathbf{1} - 1\right)\varepsilon_{kj}^{\parallel}\right]\right)$$

Special Case I: $\Pr_F(\text{EPP})$ **for $\perp$ Workflow**

$\boldsymbol{B} = \boldsymbol{I}$. Each agent performs one monitoring function in series with other agents. Each stage presents a *single point of failure:*

$$\Pr_F(\text{EPP}) = 1 - \prod_{k=1}^{M}\left(1 - p_k^{\perp}\right),$$

where Agent k works on function k

Special Case II: $\Pr_F(\text{EPP})$ **for Peak $\parallel$ Workflow**

$\boldsymbol{B} = \mathbf{1}\mathbf{1}^{\mathrm{T}}$. M agents split efforts between all monitoring functions (this is maximum parallelization):

$$\Pr_F(\text{EPP}) = 1 - \prod_{k=1}^{M}\left(1 - \prod_{j=1}^{M}\left[p_{kj}^{\perp} + (M-1)\varepsilon_{kj}^{\parallel}\right]\right)$$

Computational System to Estimate EPP Failure Rate (2/3)

$$\Pr_F(\mathrm{EPP}) = 1 - \prod_{k=1}^{\mathrm{size}(\boldsymbol{B},2)} \left(1 - \prod_{j=1}^{\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_k} \left[p_{kj}^{\perp} + \left(\hat{\boldsymbol{e}}_j^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \mathbf{1} - 1\right)\varepsilon_{kj}^{\parallel}\right]\right)$$

Special Case I: $\Pr_F(\mathrm{EPP})$ for ⊥ Workflow

$\boldsymbol{B} = \boldsymbol{I}$. Each agent performs one monitoring function in series with other agents. Each stage presents a *single point of failure* (the ⊥ baseline):

$$\Pr_F^{\perp}(\mathrm{EPP}) = 1 - \prod_{k=1}^{M}\left(1 - p_k^{\perp}\right),$$ where Agent k works on function k

Special Case II: $\Pr_F(\mathrm{EPP})$ for Peak ∥ Workflow

$\boldsymbol{B} = \mathbf{1}\mathbf{1}^{\mathrm{T}}$. M agents split efforts between all monitoring functions (the ∥ baseline):

$$\Pr_F^{\parallel}(\mathrm{EPP}) = 1 - \prod_{k=1}^{M}\left(1 - \prod_{j=1}^{M}\left[p_{kj}^{\perp} + (M-1)\varepsilon_{kj}^{\parallel}\right]\right) \to 1, \text{as } \varepsilon_{kj}^{\parallel} \to p_{kj}^{\perp}/(M-1)$$

Computational System to Estimate EPP Failure Rate (3/3)

$$\Pr_F(\mathrm{EPP}) = 1 - \prod_{k=1}^{\mathrm{size}(\boldsymbol{B},2)} \left(1 - \prod_{j=1}^{\mathbf{1}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \hat{\boldsymbol{e}}_k} \left[p_{kj}^{\perp} + \left(\hat{\boldsymbol{e}}_j^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \mathbf{1} - 1\right)\varepsilon_{kj}^{\parallel}\right]\right)$$

Special Case III: $\Pr_F(\mathrm{EPP})$ **for an "Army of One" (the AO² Model)**

$\boldsymbol{B} = \mathbf{1}^{\mathrm{T}}$. One agent performs every monitoring function in series. Each stage presents a *single point of failure*, and their effort is spilt between all monitoring functions

$$\Pr_F^{\mathbf{1}}(\mathrm{EPP}) = 1 - \prod_{k=1}^{M} \left(1 - \left(p_k^{\perp} + (M-1)\varepsilon_k^{\parallel}\right)\right),$$

where Agent k works on function k,

$\varepsilon_k^{\parallel} < p_k^{\perp}/(M-1)$

Optimal EPP Agent Network Topologies (Graphs)

Maximize EPP Success Probability

Computational System to Minimize EPP Failure Rate (1)

Task switching efficiency losses suggest that ∥ workflow is not unconditionally superior to baseline ⊥ workflow. We can show that certain parallel-processing failure probabilities bound any *minimum failure probability solutions*, if they exist.

Theorem: if efficacy loss $\varepsilon^{\parallel}_{kj} > \varepsilon_{\mathrm{cr}}$, for some j and $\varepsilon_{\mathrm{cr}}$, a matrix $\boldsymbol{B}$ exists such that the probability of *parallel workflow failure* exceeds that of *series workflow* (proof is easy).

We therefore seek a matrix $\widehat{\boldsymbol{B}}$ that reduces $\Pr_F(\mathrm{EPP})$ relative to this $\Pr^{\perp}_F(\mathrm{EPP})$. Specifically:

To find: $\min_{\boldsymbol{B}}\{\Pr_F(\mathrm{EPP})\} = \Pr_F(\mathrm{EPP})\Big|_{\widehat{\boldsymbol{B}}} \leq \Pr^{\perp}_F(\mathrm{EPP}) \leq \lim_{\varepsilon^{\parallel}_j \to \frac{1-p^{\perp}_{kj}}{M-1}} \Pr^{\parallel}_F(\mathrm{EPP}) = 1\,,$ **we solve:**

$$\min_{\boldsymbol{B}}\{\Pr_F(\mathrm{EPP})\} = 1 - \max_{\boldsymbol{B}}\left\{\prod_{k=1}^{\mathrm{size}(\boldsymbol{B},2)}\left(1 - \prod_{j=1}^{\boldsymbol{1}^{\mathrm{T}}\cdot\boldsymbol{B}\cdot\hat{\boldsymbol{e}}_k}\left[p^{\perp}_{kj} + \left(\hat{\boldsymbol{e}}^{\mathrm{T}}_j\cdot\boldsymbol{B}\cdot\boldsymbol{1} - 1\right)\varepsilon^{\parallel}_{kj}\right]\right)\right\}$$

Our solution algorithm first finds $\Pr^{\perp}_F(\mathrm{EPP})$ and $\Pr_F(\mathrm{EPP})|_{\boldsymbol{B}\neq\widehat{\boldsymbol{B}}}$, to then efficiently find $\widehat{\boldsymbol{B}}$.